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Phase compensating effect in left-handed materials

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Abstract

Based on the difference between negative refraction and negative refractive index, the phase compensating effect was proposed to distinguish negative refraction in left-handed materials (LHMs) and photonic crystals (PCs). With this effect, perfect lens (PL) of LHM in which both propagating and evanescent waves contribute to the image could be well understood, which is differed from superlens made of PCs. Furthermore, a 1D periodic structure consisted of ordinary materials and LHMs was predicted to realize higher and wider bandgaps than the 1D conventional photonic crystal due to phase compensating effect.

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Keywords: Phase compensating effect; Left-handed materials; Photonic crystals

1. Introduction

An unusual material with both permittivity and permeability simultaneously negative was theoretically investigated by Veselago in 1968 [1]. This type of material is called left-handed material (LHM) because when an electromagnetic (EM) plane wave propagate in it, the direction of Poynting vector ($\vec{E} \times \vec{H}$) will be opposite to that of wavevector (k) so that k, E, H form a left-handed set of vectors. Some unusual electrodynamic properties of LHM such as inverse Snell effect (negative refraction), reverse Doppler shift and reverse Cerenkov radiation were proposed. Based on LHM, a novel device, "perfect lens" (PL) was proposed by Pendry in which light transmits to free space without loss [8]. Recently, LHM in the microwave ranges has been predicted by Pendry et al. [2–4] and realized by Smith et al. [5–7] by constructing two-dimensional arrays consisted of split-ring resonators and wires, showing the negative refraction. On the other hand, photonic crystals (PCs) have been used to realize negative refraction which was proposed to be even to optical frequency region [9–11], showing great potential applications.

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2. The difference of negative refraction between LHMs and PCs

In this Letter, we will attempt to clarify the difference of negative refraction between LHMs and PCs (negative refraction in the lowest band). It is well known that the refractive index is responsible to the phase velocity of EM wave and the refraction is related to the group velocity. So a clear awareness should be hold that negative refraction is not equal to negative refractive index. Fig. 1 shows the difference of negative refraction between LHMs and PCs. In LHMs, there is a negative refractive index $n_{\mathrm{LHM}} = k_{\mathrm{LHM}}/k_0$. In this case, the important property is that the direction of phase velocity or wavevector k_{LHM} is opposite to the direction of group velocity or Poynting vector S_{LHM} (Fig. 1(a)), so it is the negative refractive index that causes the negative refraction in LHMs. However, the negative refraction in PCs is only resulted by intense scattering near the Brillouin zone boundaries, and its refractive index, $n_{\mathrm{PC}} = k_{\mathrm{PC}}/k_0$ is positive. Due to intense scattering, the direction of the group velocity or Poynting vector S_{PC} is divergent from that of the wavevector k_{PC} (Fig. 1(b)). Because of these two different mechanisms, there are many different phenomena between LHMs and PCs. Here our focus on negative refraction in PCs is the negative refraction in the lowest band of two-dimensional square PC.

3. Phase compensating effect in LHMs

There are two EM wave forms, one is the propagating wave

$$k_z = +\sqrt{\omega^2 c^{-2} - k_x^2 - k_y^2}, \quad \omega^2 c^{-2} > k_x^2 + k_y^2, \qquad (1)$$

the other is the evanescent wave

$$k_z' = +i\sqrt{k_x^2 + k_y^2 - \omega^2 c^{-2}}, \quad \omega^2 c^{-2} < k_x^2 + k_y^2. \qquad (2)$$

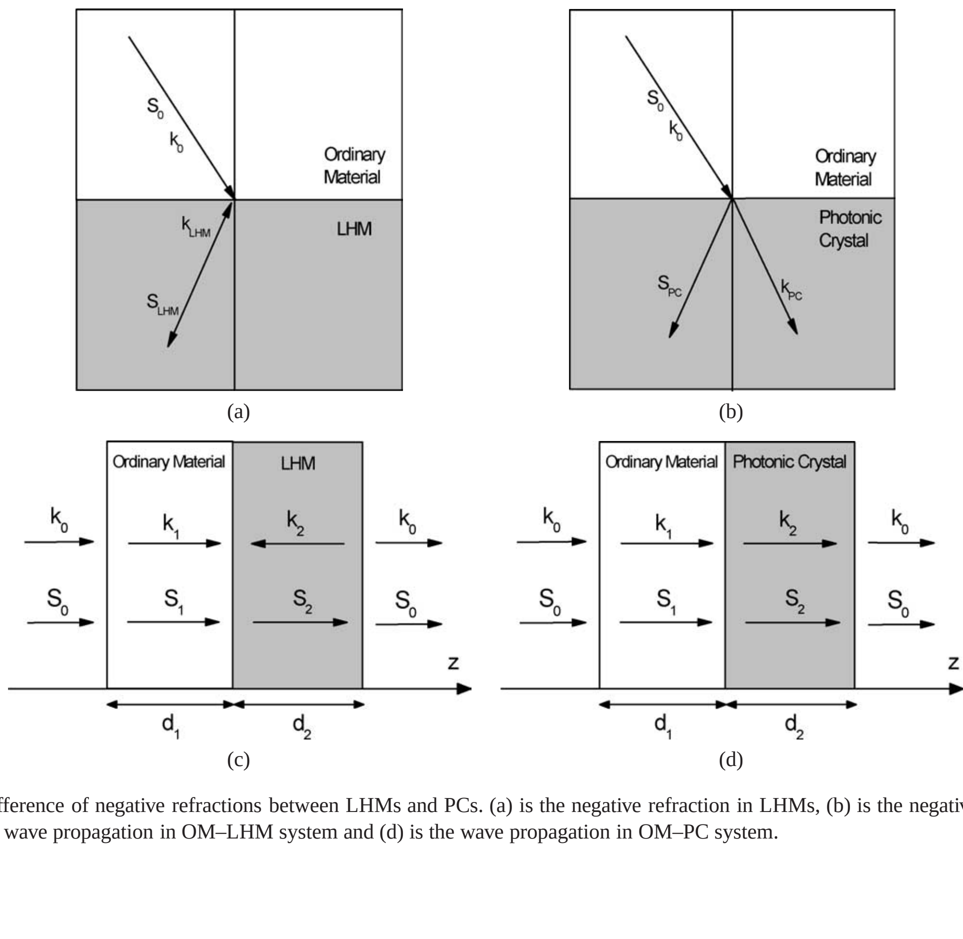

Fig. 1. The difference of negative refractions between LHMs and PCs. (a) is the negative refraction in LHMs, (b) is the negative refraction in PCs, (c) is the wave propagation in OM–LHM system and (d) is the wave propagation in OM–PC system.

Only could the wave propagate in a medium when

$$\omega^2 c^{-2} > k_x^2 + k_y^2. \tag{3}$$

Consider a two-layer slab as Fig. 1(c) and (d), one is consisted of ordinary material (OM) and LHM (Fig. 1(c)) and the other is OM and PC (Fig. 1(d)), in which the impedances of both regions are the same as the one of free space ($Z_1 = Z_2 = Z_0$), resulting in the perfect transmission of EM wave in both materials.

In the OM–LHM case, the indexes of the two-layer are

$$n_1 = \sqrt{\varepsilon_1 \mu_1}, \qquad n_{\mathrm{LHM}} = -\sqrt{\varepsilon_{\mathrm{LHM}} \mu_{\mathrm{LHM}}}, \tag{4}$$

respectively. EM waves enter the system from the left surface of the first layer and exit from the right surface of the second layer. In the first layer, the wavevectors of two kinds of waves, k_1 and k_1' are parallel to the direction of Poynting vector (S_1). The phase difference in the first layer is $n_1 \frac{\omega}{c} d_1$. That in propagating waves is

$$k_{1z} d_1 = +\sqrt{n_1^2 \omega^2 c^{-2} - k_{1x}^2 - k_{1y}^2}\, d_1, \tag{5}$$

and for evanescent waves, that is

$$k_{1z}' d_1 = +i\sqrt{k_{1x}^2 + k_{1y}^2 - n_1^2 \omega^2 c^{-2}}\, d_1. \tag{6}$$

In the second layer, the wavevectors of two kinds of waves (k_{LHM} and k_{LHM}') are anti-parallel to the direction of Poynting vector (S_{LHM}). The phase difference of the this layer is $n_{\mathrm{LHM}} \frac{\omega}{c} d_2$, for both propagating waves and evanescent waves, which are

$$k_{\mathrm{LHM}z} d_2 = -\sqrt{n_{\mathrm{LHM}}^2 \omega^2 c^{-2} - k_{\mathrm{LHM}x}^2 - k_{\mathrm{LHM}y}^2}\, d_2, \tag{7}$$

$$k_{\mathrm{LHM}z}' d_2 = -i\sqrt{k_{\mathrm{LHM}x}^2 + k_{\mathrm{LHM}y}^2 - n_{\mathrm{LHM}}^2 \omega^2 c^{-2}}\, d_2, \tag{8}$$

respectively. Because the impedances of the two layers are the same as the one of free space ($Z_1 = Z_2 = Z_0$), there is no reflection at this system's interfaces. Therefore, the total phase differences of this system are $n_1 \frac{\omega}{c} d_1 + n_{\mathrm{LHM}} \frac{\omega}{c} d_2$ for both propagating waves and evanescent waves, which are

$$\begin{aligned} k_{1z} d_1 + k_{\mathrm{LHM}z} d_2 &= +\sqrt{n_1^2 \omega^2 c^{-2} - k_{1x}^2 - k_{1y}^2}\, d_1 \\ &\quad - \sqrt{n_{\mathrm{LHM}}^2 \omega^2 c^{-2} - k_{\mathrm{LHM}x}^2 - k_{\mathrm{LHM}y}^2}\, d_2, \end{aligned} \tag{9}$$

$$\begin{aligned} k_{1z}' d_1 + k_{\mathrm{LHM}z}' d_2 &= +i\sqrt{k_{1x}^2 + k_{1y}^2 - n_1^2 \omega^2 c^{-2}}\, d_1 \\ &\quad - i\sqrt{k_{\mathrm{LHM}x}^2 + k_{\mathrm{LHM}y}^2 - n_{\mathrm{LHM}}^2 \omega^2 c^{-2}}\, d_2, \end{aligned} \tag{10}$$

respectively. Because the signs of the indexes are reverse and the directions of the wavevectors in two layers are opposite, the total phase difference would be smaller than the one of the first layer. Hence there is a phase compensating effect in LHM. For propagating waves, LHM acts as a phase compensator in this system [12]. It is clearly showed if both length and absolute value of refractive index of two slabs in Eq. (9) are the same, the phase difference is zero. Meanwhile for evanescent waves, LHM reduces the decay in amplitude and acts as an amplitude compensator due to the imaginary part of evanescent waves in the wave equation. The total effect is to compensate the amplitude reduction in OM. It is interesting that the evanescent amplitude would be the same as the origin when the length and absolute value of refractive index of the first slab are the same as those of the second one in Eq. (10). So evanescent waves also can be transmitted and waves in LHM can surpass the image limitation due phase compensating effect.

However, in the OM–PC case, the indexes of the two-layer are

$$n_1 = \sqrt{\varepsilon_1 \mu_1}, \qquad n_{\mathrm{PC}} = \sqrt{\varepsilon_{\mathrm{PC}} \mu_{\mathrm{PC}}}, \tag{11}$$

respectively. The signs of the indexes and the directions of the wavevectors in two layers are the same, so PCs cannot be a phase compensator for propagating waves while the evanescent waves will decay exponentially. But near the resonant frequency, it is also possible to obtain transmission amplitudes for evanescent waves [13].

4. A perfect lens based on LHM

A perfect lens (PL) based on LHM is proposed in Ref. [8] as Fig. 2(a), which is a slab of LHM with $\varepsilon_{\mathrm{PL}} = -1$ and $\mu_{\mathrm{PL}} = -1$. The refractive index is negative with $n_{\mathrm{PL}} = -1$ and the wavevector in PL is

$$k_{\mathrm{PL}} = \frac{n_{\mathrm{PL}}}{n_0} k_0 = -k_0. \tag{12}$$

Due to negative refractions, there are two focusing image, one in PLs interior and the other at PLs right region. Another important character is that the impedance of PL is equal to that of free space,

$$Z = \sqrt{\frac{\mu_0 \mu_{\mathrm{LHM}}}{\varepsilon_0 \varepsilon_{\mathrm{LHM}}}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 1, \tag{13}$$

which assures there is no reflection at every interface. This makes all propagating waves contribute to the image. From Fig. 2(a), the light distances of EM waves in free space and PL are the same and can be defined as d. For propagating waves, the phase difference between the object and the image is $k_{0z}d + k_{\mathrm{PL}z}d = 0$, which means that there is no information difference between the object and the image. On the other hand the phase difference of evanescent waves is also $k'_{0z}d + k'_{\mathrm{PL}z}d = i0$, which means that the amplitude of evanescent waves does not change when they propagate so that PL could make use of all evanescent waves. Hence all information of the object, no matter propagating and evanescent waves, can be displayed in the image. Fig. 2(b) is the phase varying simulation with an infinite PL that depicts the phase variation from a dipole source at PLs left region. Due to the phase compensating effect, every point in PL has an identical phase with its symmetrical point in PLs left and right regions. Both images in PL and PLs right region are equal to the source at PLs left region. So based on phase compensating effect, PLs mechanism could be easily understood.

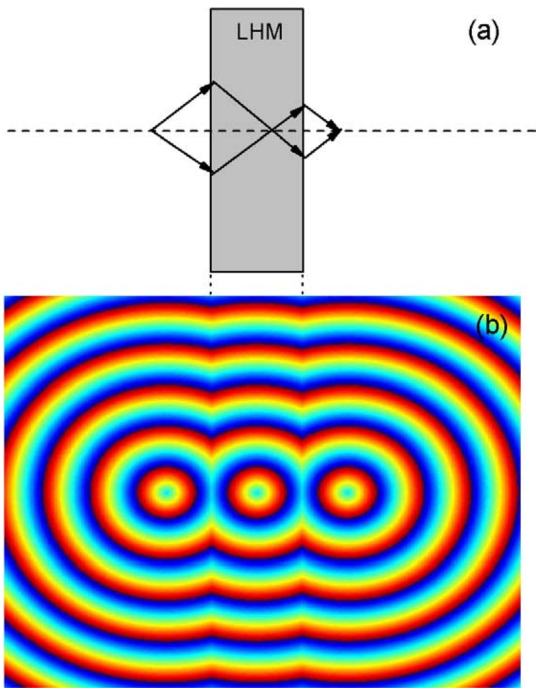

Fig. 2. The perfect lens. (a) is the schematic of the perfect lens and (b) is the phase variation in an infinite perfect lens system.

Recently imaging by another flat lens using negative refraction of PCs was realized in experiment [15]. This type of lens called superlens that can obtain subwavelength image is also discussed [13]. However, with positive refractive index, there is no phase compensating effect, so the same perfect image as its source might not be obtained. Here the 'perfect' means not only the image of evanescent waves but also the identity of the amplitude and phase.

5. 1D periodic structure consisted of OMs and LHMs

Consider EM wave propagation in one-dimensional (1D) system composed of periodic arrays of ordinary materials and LHMs (Fig. 3(a)), where we define d_1 and d_2 as the length of the ordinary material slab and LHM slab, respectively. Then the lattice constant or the length of unit cell is $d = d_1 + d_2$.

Maxwell's equations for the 1D system can reduce to the following wave equation in every slab:

$$\frac{\partial E(z,t)}{\partial z^2} = \frac{\varepsilon(z)\mu(z)}{c^2} \frac{\partial^2 E(z,t)}{\partial t^2}. \tag{14}$$

By assuming the time dependence

$$E(z,t) = E(z)\exp(-i\omega t), \tag{15}$$

Eq. (14) can be written as

$$\frac{\partial^2 E(z)}{\partial z^2} + \frac{\omega^2}{c^2}\varepsilon(z)\mu(z)E(z) = 0. \tag{16}$$

When the slabs are positioned periodically such that $\varepsilon(x+d) = \varepsilon(x)$ and $\mu(x+d) = \mu(x)$, the solution of equation has a similar form as Bloch wave

$$E(z+d) = \exp(ikd)E(z), \tag{17}$$

where k is known as the crystal momentum, or Bloch vector. In each three region, equation yields the usual plane wave solutions with arbitrary coefficients, which we can describe as

$$\begin{aligned}
&\text{region I}, && E_{\mathrm{I}}(z) = A\exp(ik_1z) + B\exp(-ik_1z),\\
&\text{region II}, && E_{\mathrm{II}}(z) = C\exp(ik_2z) + D\exp(-ik_2z),\\
&\text{region III}, && E_{\mathrm{III}}(z) = A'\exp(ik_1z) + B'\exp(-ik_1z),
\end{aligned} \tag{18}$$

where $k_1 = \frac{n_1\omega}{c}$ $(n_1 > 0)$ and $k_2 = \frac{n_2\omega}{c}$ $(n_2 < 0)$. Then the condition of translational invariance expressed by equation requires that

$$\begin{bmatrix} A'\exp(ik_1d) \\ B'\exp(-ik_1d) \end{bmatrix} = \exp(ikd)\begin{bmatrix} A \\ B \end{bmatrix}. \tag{19}$$

With assuming the permeabilities of system as $\mu_1 = -\mu_2 = 1$ for better comparison with the 1D conventional photonic crystal in which the permeability is set as 1 [14], the refractive index becomes the reciprocal of impendence so that the refractive index could be considered instead of impendence [16,17] to calculate the bandstructure. Then, we have the boundary continuous conditions

$$\begin{aligned}
&E_{\mathrm{I}}(z) = E_{\mathrm{II}}(z)|_{z=-d_1/2},\\
&E'_{\mathrm{I}}(z) = -E'_{\mathrm{II}}(z)|_{z=-d_1/2},\\
&E_{\mathrm{II}}(z) = E_{\mathrm{III}}(z)|_{z=-d_1/2},\\
&E'_{\mathrm{II}}(z) = -E'_{\mathrm{III}}(z)|_{z=d_1/2}.
\end{aligned} \tag{20}$$

From Eq. (20) we can relate the traveling-wave coefficients of region I to those of region III by a matrix equation:

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = T\begin{bmatrix} A \\ B \end{bmatrix}, \tag{21}$$

where

$$\begin{aligned}
T_{11} &= \exp(-ik_1d_1)\\
&\quad\times\left[\cos(k_2d_1) - i\left(\frac{n_1}{n_2} + \frac{n_2}{n_1}\right)\frac{\sin(k_2d_1)}{2}\right],\\
T_{12} &= i\left(\frac{n_1}{n_2} - \frac{n_2}{n_1}\right)\frac{\sin(k_2d_1)}{2},\\
T_{21} &= T_{12}^*,\\
T_{22} &= T_{11}^*.
\end{aligned} \tag{22}$$

Using the condition of translational invariance, we can write Eq. (18) as

$$\begin{aligned}
&\begin{bmatrix} A'\exp(ik_1d) \\ B'\exp(-ik_1d) \end{bmatrix}\\
&\quad= \begin{bmatrix} \exp(ik_1d) & 0 \\ 0 & \exp(-ik_1d) \end{bmatrix}\begin{bmatrix} A' \\ B' \end{bmatrix}\\
&\quad= \begin{bmatrix} \exp(ik_1d) & 0 \\ 0 & \exp(-ik_1d) \end{bmatrix} T\begin{bmatrix} A \\ B \end{bmatrix}\\
&\quad= \exp(ikd)\begin{bmatrix} A \\ B \end{bmatrix}.
\end{aligned} \tag{23}$$

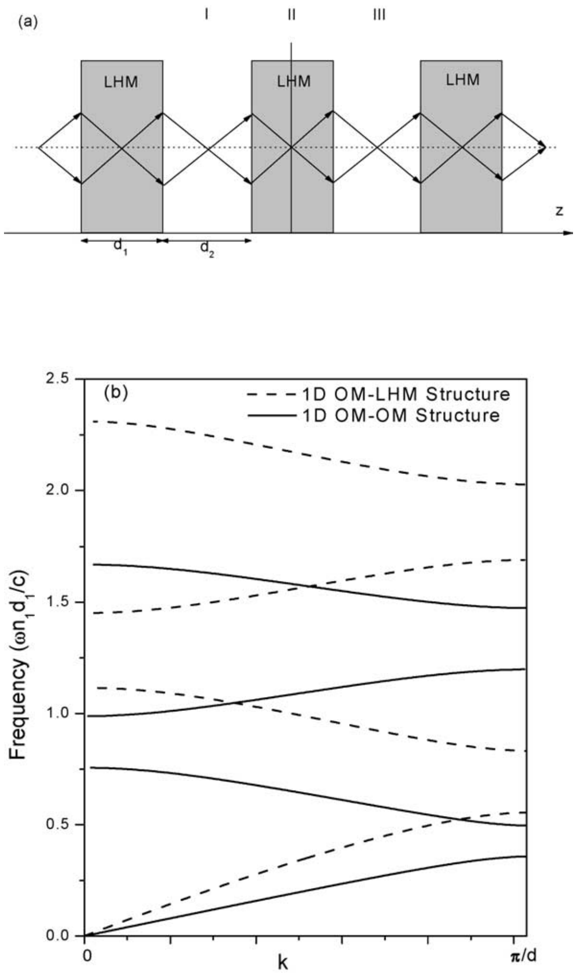

Fig. 3. A 1D periodic system composed of alternate layers. (a) is the sketch of alternate layers of ordinary materials and LHMs; (b) the dispersive relations of 1D periodic structure composed of alternate layers of ordinary materials and LHMs (dashed curves) and conventional photonic crystal (solid curves) with $|n_2| = 3|n_1|$ and $d_2 = 2d_1$.

From Eq. (23),

$$\left\{\begin{bmatrix} \exp(ik_1 d) & 0 \\ 0 & \exp(-ik_1 d) \end{bmatrix} T - \exp(ikd) I\right\} \times \begin{bmatrix} A \\ B \end{bmatrix} = 0, \tag{24}$$

where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Then we can get the dispersion relation between the frequency ω and the crystal momentum k

$$\begin{aligned} \cos k(d_1 + d_2) &= \cos\frac{n_1\omega d_1}{c}\cos\frac{n_2\omega d_2}{c} + \frac{1}{2}\left(\frac{n_1}{n_2} + \frac{n_2}{n_1}\right)\sin\frac{n_1\omega d_1}{c}\sin\frac{n_2\omega d_2}{c} \\ &= \cos\frac{|n_1|\omega d_1}{c}\cos\frac{|n_2|\omega d_2}{c} + \frac{1}{2}\left(\frac{|n_1|}{|n_2|} + \frac{|n_2|}{|n_1|}\right)\sin\frac{|n_1|\omega d_1}{c}\sin\frac{|n_2|\omega d_2}{c}. \end{aligned} \tag{25}$$

This relation is different from the 1D system composed of periodic arrays of two different ordinary materials with $n'_1 > 0$ and $n'_2 > 0$ [14], i.e., a 1D conventional photonic crystal,

$$\cos k\left(d'_1 + d'_2\right) = \cos\frac{n'_1\omega d'_1}{c}\cos\frac{n'_2\omega d'_2}{c} - \frac{1}{2}\left(\frac{n'_1}{n'_2} + \frac{n'_2}{n'_1}\right)\sin\frac{n'_1\omega d'_1}{c}\sin\frac{n'_2\omega d'_2}{c}. \tag{26}$$

From Eqs. (25) and (27), we could see if the right terms of the equation is more than 1, there will exists the stop bandgap, caused by the ratio of two different refractive indexes $|n_1|/|n_2|$ or $|n_2|/|n_1|$. Fig. 3(b) shows these two different dispersive relations described by Eqs. (25) and (27). Here the corresponding refractive indexes have the same absolute value, $n_1 = n'_1 = |n_1|$ and $-n_2 = n'_2 = |n_2|$. Compared with 1D conventional PC (solid), the periodic array including LHMs (dashed) has wider bandgaps and higher frequencies. Based on phase compensating effect, it could be understood that the phase compensation in every LHM slab leads to the crystal momentum k less than that in 1D conventional PC and the lower crystal momentum k here is corresponding to higher frequency. In another word, the nominal same k does not mean the same wave length but the longer one in OM–LHM than in OM–OM which results in increasing correspondent frequency. Therefore, compared with a 1D OM–OM periodic structure, a 1D OM–LHM one has higher and wider Bragg bandgaps with the same transmission. Based on this feature, by applying LHMs in 2D or 3D periodic structure, the full band gap in them should be effectively enhanced and enlarged. Meanwhile, the zero-$\tilde{n}$ bandgap also could be expected when the wavelength is much longer than the repeat distance of the structure including LHM [18].

For a special case in which the absolute value of the index of this system is the same, $n_1 = -n_2 = n$, and length of the slabs is $d_1 = d_2 = d/2$. Then the dispersion relation Eq. (25) can be reduced to

$$\cos k(d_1 + d_2) = \cos^2\frac{n\omega d}{2c} + \sin^2\frac{n\omega d}{2c} = 1. \tag{27}$$

The frequency ω is not related to the crystal momentum k because the crystal momentum here is zero, which is obviously caused by phase compensating effect. So the frequency ω is only related to the wavevector k_1 or k_2, and can be written as

$$\omega = \frac{ck_1}{n_1} = \frac{ck_2}{n_2} = \frac{c|k_1|}{n}. \tag{28}$$

If $n_1 = -n_2 = 1$, this system can be regarded as the juxtaposition of some identical PLs and free space, in which all waves will transmit. Therefore, through calculation in 1D structure, the same impedance with free space and phase compensating effect in LHM are the key mechanisms of PL, which results in both propagating and evanescent waves contribute to the resolution of the image.

6. Conclusions

To summarize, the difference of the negative refractive index and negative refraction was proposed to discuss the propagation of EM wave in LHMs and PCs. From the negative refractive index, phase compensating effect for propagating wave and amplitude compensating effect for evanescent wave were established in LHMs. Due to these effects both propagating and evanescent waves could propagate in LHMs and contribute to the images which result in perfect lens different from superlens realized by negative refraction of PCs. A 1D periodic structure consisted of

OMs and LHMs was calculated to show higher and wider bandgaps than 1D conventional PC with the same transmission.

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